

## Problem 2.84

[Difficulty: 2]

**2.84** The viscous boundary layer velocity profile shown in Fig. 2.15 can be approximated by a cubic equation,

$$u(y) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^3$$

The boundary condition is  $u = U$  (the free stream velocity) at the boundary edge  $\delta$  (where the viscous friction becomes zero). Find the values of  $a$ ,  $b$ , and  $c$ .

**Given:** Boundary layer velocity profile in terms of constants  $a$ ,  $b$  and  $c$

**Find:** Constants  $a$ ,  $b$  and  $c$

**Solution:**

Basic equation 
$$u = a + b \cdot \left(\frac{y}{\delta}\right) + c \cdot \left(\frac{y}{\delta}\right)^3$$

Assumptions: No slip, at outer edge  $u = U$  and  $\tau = 0$

At  $y = 0$   $0 = a$   $a = 0$

At  $y = \delta$   $U = a + b + c$   $b + c = U$  (1)

At  $y = \delta$   $\tau = \mu \cdot \frac{du}{dy} = 0$

$$0 = \frac{d}{dy} a + b \cdot \left(\frac{y}{\delta}\right) + c \cdot \left(\frac{y}{\delta}\right)^3 = \frac{b}{\delta} + 3 \cdot c \cdot \frac{y^2}{\delta^3} = \frac{b}{\delta} + 3 \cdot \frac{c}{\delta}$$

$b + 3 \cdot c = 0$  (2)

From 1 and 2  $c = -\frac{U}{2}$   $b = \frac{3}{2} \cdot U$

Hence 
$$u = \frac{3 \cdot U}{2} \cdot \left(\frac{y}{\delta}\right) - \frac{U}{2} \cdot \left(\frac{y}{\delta}\right)^3$$
 
$$\frac{u}{U} = \frac{3}{2} \cdot \left(\frac{y}{\delta}\right) - \frac{1}{2} \cdot \left(\frac{y}{\delta}\right)^3$$

